Quarkonium during the hydrodynamical phase

Clint Young

SUNY Stony Brook

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Modeling charmonium with a Langevin equation

- Heavy quark and quarkonium dynamics
- Quarkonium dynamics in sQGP as a stochastic process
- Properties of the J/ψ in sQGP

Quarkonium as an open quantum system

- The path integral approach to quantum Brownian motion
- Imaginary-time correlators

Au+Au RHIC collisions

- Langevin-with-interaction simulation of charmonium
- Recombinant production
- Cold nuclear matter effects
- Anomalous J/ψ suppression for two values of T_c

Conclusions and future work

Part I: Modeling charmonia with Langevin equations

Heavy quark and quarkonium dynamics

Heavy quark diffusion: $3\kappa = \int d^3q|\mathbf{q}|^2 \frac{d^3\Gamma}{dq^3}$

- HTL approximation at NLO (Caron-Huot and Moore)[1]:

$$\kappa = \frac{16\pi}{3}\alpha_s^2 T^3 \left(\log(1/g_s) + .07428 + 1.9026 g_s\right) \qquad (1)$$

- Drag force and diffusion from AdS/CFT (Gubser, Casalderrey-Solana and Teaney, Mia et al.) [2], [3], [4], [5]:

$$\kappa = \pi\sqrt{\lambda}T^3 \qquad (2)$$

- Phenomenology (Moore and Teaney) [6].

$Q\bar{Q}$ potential

- Lattice calculations of $Tr\left\langle W(\mathbf{x})W^\dagger(\mathbf{0})\right\rangle$ (Kaczmarek et al.) [7].
- Internal or Free Energy? (Shuryak and Zahed) [8].
- Potential models (Mocsy and Petreczky) [9].

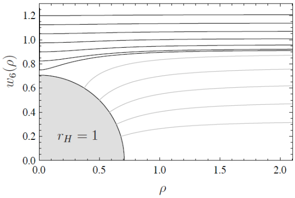
D_{HQ} vs. quarkonium diffusion

Quarkonium $\neq$ two heavy quarks!

First AdS/CFT calculations for quarkonium suggested *zero drag*, only influence of the thermal medium from a "hot wind"

Dusling, ..., Young: Fluctuations on D7 in $AdS_5 \times S_5$ dual to effective theory for dipoles

Momentum diffusion suppressed by factor of $1/N_c^2$, smaller than perturbative $\mathcal{N} = 4$ prediction by a factor of 4. Exact opposite of heavy quark situation!



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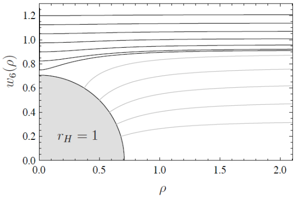

However, this treatment only valid when $E_B >> T$

Appropriate for Υ, inappropriate for J/ψ

J/ψ dynamics at RHIC somewhere between "photoelectric effect" and "Rayleigh scattering"

Quarkonium dynamics in sQGP as a stochastic process

When M_{HQ} is sufficiently larger than T, the dynamics of each heavy quark can be described by

$$\frac{dp_i}{dt} = -\eta p_i + \xi_i - \nabla_i U, \tag{3}$$

where

$$\langle \xi_i(t)\xi_j(0)\rangle = \kappa\delta_{ij}\delta(t). \tag{4}$$

Requiring thermalization to temperature T yields the Einstein relation between noise and dissipation:

$$\eta = \frac{\kappa}{2MT}. \tag{5}$$

Ito integration

When a stochastic process satisfies a relaxed version of continuity, it can be integrated:

$$p^i_{n+1} = (1 - \eta\Delta t)p^i_n + \xi^i_n \Delta t, \tag{6}$$

where ξ^i_n is now selected from a Gaussian distribution with expectation value κ, thanks to the central limit theorem (the sum of a sufficiently large number of random variables will approach a Gaussian distribution).

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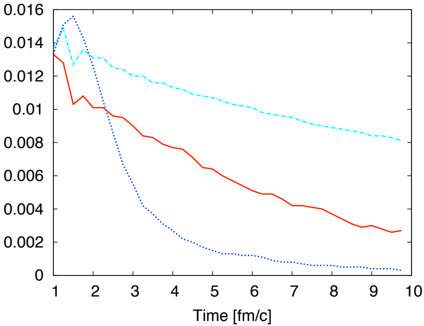
The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement, amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason.

Sir Francis Galton, 1889

Evolution of an ensemble of $Q\bar{Q}$ pairs in sQGP

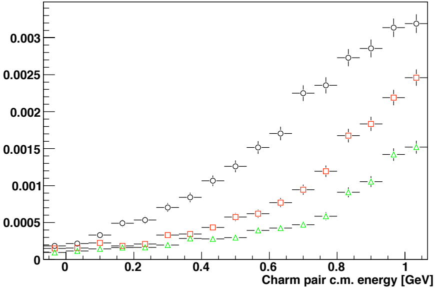
The probability for a $Q\bar{Q}$ pair to be bound as a function of time:

- **Green:** $2\pi TD_c = 1.5$
- **Red:** $2\pi TD_c = 3.0$
- **Blue:** $2\pi TD_c = 1.5$, no $Q\bar{Q}$ interaction



Quasi-equilibrium of $Q\bar{Q}$-pair ensemble

- **Black:** Ensemble after $\tau = 2\ \mathrm{fm/c}$
- **Red:** Ensemble after $\tau = 3\ \mathrm{fm/c}$
- **Green:** Ensemble after $\tau = 10\ \mathrm{fm/c}$



Only the normalization changes after the initial thermalization $\sim 2\ \mathrm{fm/c}$.

Summary of $Q\bar{Q}$ in sQGP

- Thermalization in momentum space relatively fast, spatial diffusion relatively slow.
- The $Q\bar{Q}$-potential can greatly enhance the survival probability.
- Quasi-equilibrium forms: relative abundances predicted by Boltzmann factors.

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An explanation for J/ψ survival at RHIC?

Part II: Quarkonium as an open quantum system

The reduced density matrix

Imagine a single degree of freedom minimally coupled to a bath:

$$
\begin{aligned}
L &= \frac{1}{2}M\dot{x}^2 - V(x) \\
&+ \frac{1}{2}\sum_i m_i \dot{R}_i^2 - \frac{1}{2}\sum_i m_i \omega_i^2 R_i^2 \\
&- \sum_i C_i x R_i .
\end{aligned}
\tag{7}
$$

The *reduced* density matrix

$$
\begin{aligned}
\rho_{red}(x, x', \beta) &= \int dR_i \rho(x, R_i, x', R_i, \beta) \\
&= \int Dx \exp(-\int_0^\beta d\tau [\frac{1}{2}M\dot{x}^2 + V(x) \\
&- \sum_i \frac{C_i^2}{2m_i\omega_i \sinh(\omega_i\beta/2)} x(\tau) \int_0^\tau ds\, x(s) \cosh(\omega_i(\tau - s - \beta/2))])
\end{aligned}
\tag{8}
$$

Caldeira and Leggett, 1983

Intuitively, when the proper infinite limit is taken for the bath, the dynamics for the heavy particle may be dissipative. The density of states

$$C^2(\omega)\rho_D(\omega) = \begin{cases} \frac{2m\eta\omega^2}{\pi} & \text{if } \omega < \Omega \\ 0 & \text{if } \omega > \Omega \end{cases} \tag{9}$$

yields Langevin dynamics when $\Omega \to \infty$, with η the usual drag coefficient.

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The reduced density matrix becomes, after integrating by parts and renormalizing...

The reduced density matrix for an open system

$$\begin{aligned}\rho_{red}(x_i, x_f, \beta) &= \int_{x(0)=x_i}^{x(\beta)=x_f} \mathcal{D}x \;\exp\Big\{ - S_S^E[x] \\ &- \frac{\eta}{2\pi}(x_i - x_f)^2 \left[\gamma_E + \ln\left(\frac{\eta\beta}{\pi M}\right)\right] \\ &+ \frac{\eta}{\pi}(x_i - x_f)\int_0^\beta d\tau\; \dot{x}(\tau)\, \mathrm{lnsin}\left(\frac{\pi\tau}{\beta}\right) \\ &+ \frac{\eta}{\pi}\int_0^\beta d\tau \int_0^\tau ds\; \dot{x}(\tau)\dot{x}(s)\, \mathrm{lnsin}\left(\frac{\pi(\tau - s)}{\beta}\right) \Big\}. \quad (10)\end{aligned}$$

Imaginary-time correlators

Correlation functions for heavy-heavy composite operators calculated on the lattice:

$$G(\tau) = \int d^3x \, \langle J^\mu(\mathbf{x}, \tau) J_\mu(\mathbf{0}, 0) \rangle \,, \tag{11}$$

$$J^\mu(\mathbf{x}, \tau) = \bar{q}(\mathbf{x}, \tau) \gamma^\mu q(\mathbf{x}, \tau). \tag{12}$$

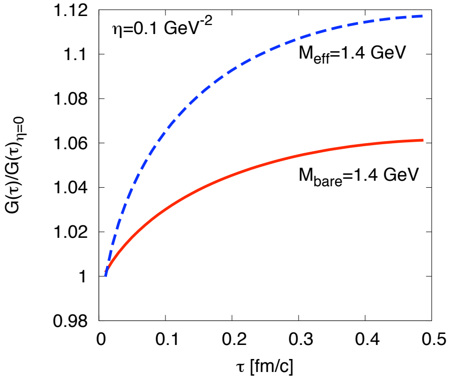
The leading term in the limit $M >> T$ is given by

$$\langle 0 | J^\mu(\mathbf{x}, \tau) J_\mu(\mathbf{0}, 0) | 0 \rangle = \langle \mu, \mathbf{x}; \tau | \mu, \mathbf{0}; 0 \rangle_{\mathrm{light}} \,. \tag{13}$$

The RHS of 13: first-quantized form

The effect of dissipation on $G_{rec}(\tau)$

- $\eta = 0.1\mathrm{GeV}^{-1}$,
- $T = 1.2T_c$.



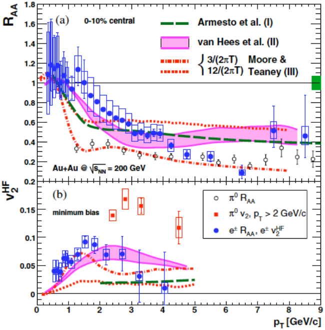
Part III: Langevin-with-interaction simulation of charmonium in Au+Au RHIC Collisions

Langevin-with-interaction simulation of charmonium

LO PYTHIA event generation

2+1-dimensional hydrodynamical simulation of the plasma phase

Langevin+interaction evolution of the $c\bar{c}$ pairs



Another consideration: recombinant production

- With many hard processes per collision, the possibility of recombinant production needs to be considered.
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Facts about charm production at the RHIC:

- On average 18 pairs produced in the most central Au+Au collisions.
- Only 5.5% of charm quarks produced are "neighbors" (close enough to form a bound state) with a single anti-quark. Only an additional 0.2% have more than one neighbor.

Boltzmann factors for an N $c\bar{c}$ system

For an event with N $c\bar{c}$ pairs, there are $N!$ possible pairings, each with with probability (in thermal equilbrium but obviously not *chemical* equilibrium)

$$P(\sigma) = \frac{1}{\mathcal{Z}} \exp(-E_\sigma / T_c),\ \mathcal{Z} = \sum_{\sigma \in S_N} \exp(-E(\sigma)/T_c). \qquad (14)$$

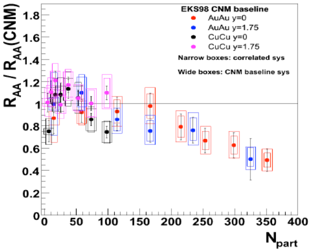
Thermal equilibration *not* assumed; only pairings of unbound quarks considered.

Facts from last slide considerably simplify this calculation; at the LHC the Metropolis algorithm is necessary.

J/ψ suppression and CNM effects

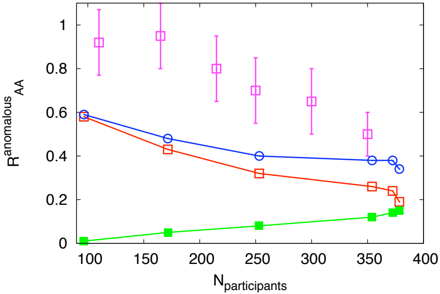
Our results compared with latest CNM effect analysis, where shadowing and breakup are treated separately

Successful in explaining greater supression at forward rapidity as a CNM effect

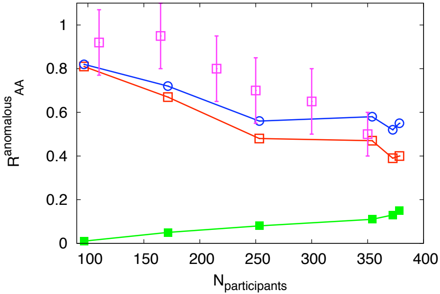


Anomalous J/ψ suppression for two values of T_c

For $T_c = 165\ \mathrm{MeV}$:

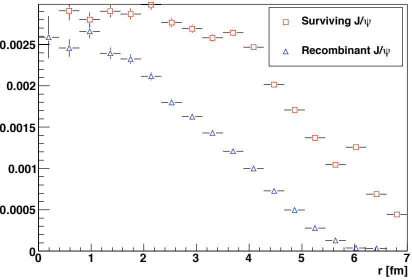


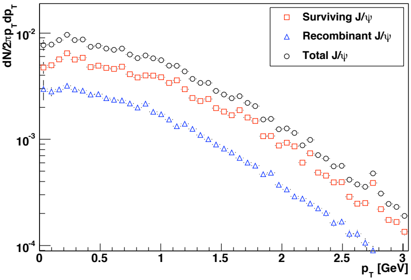
For $T_c = 190\ \mathrm{MeV}$:



Can differential p_T yields differentiate between the two components?

The surviving component in the periphery of the transverse plane, the recombinant peaked in the center.





Conclusions and future work

A model for J/ψ suppression and charmonium thermalization at the RHIC proposed which is *consistent with heavy quark dynamics in strongly-coupled gauge theory*.

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Calculate surviving and recombinant yields at the LHC

Extract spectral functions for quarkonium correlators with the maximal entropy method, *decouple "disassociation rates" in this model from hydrodynamics simulations*

Other observables at the RHIC (invariant mass spectrum of dielectrons, bottomonium)?

What more can AdS/CFT tell us about quarkonium?

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Thanks!

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